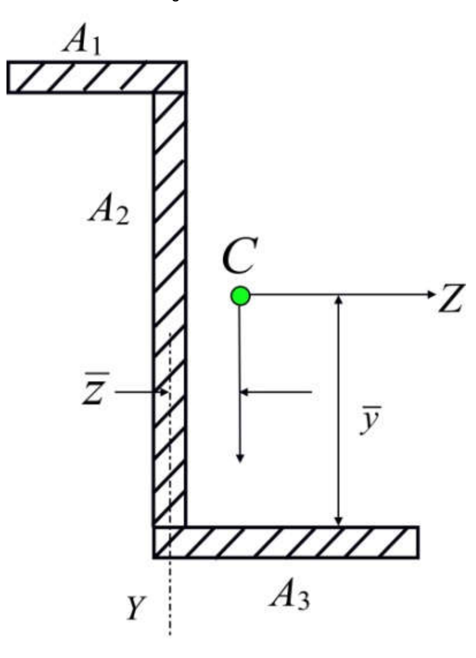
Advanced Strength and Applied Elasticity (4th Edition)

Chapter 5, Problem 21P Bookmark Show all steps: ON

Step-by-step solution

Step 1 of 25

Draw the schematic diagram of the z-section.



Comment

Step 2 of 25

Calculate of the segment 1.

$A_1 = b_1 \times h_1$

Here, b_1 is with of segment 1 and h_1 is thickness of segment 1.

Substitute $15.5\,\text{mm}$ for h_1 and $250\,\text{mm}$ for b_1.

$$A_1 = 15.5 \times 75 = 1162.5\,\text{mm}^2$$

Comments (1)

Step 3 of 25

Calculate the centroid of segment 1 in the z-direction.

$\overline{z}_1 = -\frac{b_1}{2}$

Here, $\overline{z}_1$ centroid of segment 1 in the z-direction.

Substitute $75\,\text{mm}$ for b_1.

$$\overline{y}_1 = -\left(\frac{75\text{mm}}{2}\right) = -37.5\,\text{mm}$$

Comment

Step 4 of 25

Calculate the area of segment 2.

$A_2 = b_2 \times h_2$

Here, b_2 is width of segment 2 and h_2 is height of segment 2.

Substitute $250\,\text{mm}$ for h_2 and $9.5\,\text{mm}$ for b_2.

$$A_2 = 9.5 \times 250 = 2375\,mm^2$$

Comment

Step 5 of 25

Calculate centroid of segment 2 in the z-direction.

$\overline{z}_2 = 0$

Here, $\overline{z}_2$ is centroid of segment in the z-direction.

Comment

Step 6 of 25

Calculate the area of segment 3

$A_3 = b_3 \times h_3$

Here, b_3 is width of segment 3 and h_3 is thickness of segment 3.

Substitute $125\,\text{mm}$ for b_3 and $9.5\,\text{mm}$ for h_3.

$$A_3 = 125 \times 9.5 = 1187.5\,\text{mm}^2$$

Comment

Step 7 of 25

Calculate the centroid of segment 3 in z-direction

$\overline{z}_3 = \frac{b_3}{2}$

Here, $\overline{z}_3$ is the centroid of segment 3 in z-direction.

Substitute 125 mm for b_3.

$$\overline{z}_3 = \frac{125}{2} = 62.5\text{mm}$$

Comment

Step 8 of 25

Calculate the centroid of segment 1 in the y-direction.

$\overline{y}_1 = -250\,\text{mm}$

Here, $\overline{y}_1$ is the centroid of segment 1 in the y-direction.

Calculate the centroid of segment 2 in the y-direction.

$\overline{y}_2 = 0\,\text{mm}$

Here, $\overline{y}_2$ is the centroid of segment 2 in the y-direction.

Calculate the centroid of segment 3 in the y-direction.

$\overline{y}_3 = 250\,\text{mm}$

Here, $\overline{y}_3$ is the centroid of segment 2 in the y-direction.

Comment

Step 9 of 25

Calculate the location of centroid about y-axis.

$\overline{y} = \frac{A_1\overline{y}_1 + A_2\overline{y}_2 + A_3\overline{y}_3}{A_1 + A_2 + A_3}$

Here, A_1 is the area of segment 1, A_2 is area of segment 2 and A_3 is area of segment 3.

Substitute $1162.5\,\text{mm}^2$ for A_1, $-250\,\text{mm}$ for $\overline{y}_1$, $2375\,\text{mm}^2$ for A_2, $0\,\text{mm}$ for $\overline{y}_2$, $1187.5\,\text{mm}^2$ for A_3 and $250\,\text{mm}$ for $\overline{y}_3$.

$$\begin{aligned}\overline{y} &= \frac{(1162.5)(-250)+(2375)(0)+(1187.5)(-250)}{1162.5+2375+1187.5}\\ &= \frac{-587500}{4725}\\ &= -124.339\,\text{mm}\end{aligned}$$

Comments (1)

Step 10 of 25

Calculate the location of centroid about z-axis.

$\overline{z} = \frac{A_1\overline{z}_1 + A_2\overline{z}_2 + A_3\overline{z}_3}{A_1 + A_2 + A_3}$

Here, A_1 is the area of segment 1, A_2 is area of segment 2 and A_3 is area of segment 3.

Substitute $1162.5\,\text{mm}^2$ for A_1, $-37.5\,\text{mm}$ for $\overline{z}_1$, $2375\,\text{mm}^2$ for A_2, $0\,\text{mm}$ for $\overline{z}_2$, $1187.5\,\text{mm}^2$ for A_3 and $62.5\,\text{mm}$ for $\overline{z}_3$.

$$\begin{aligned}\overline{z} &= \frac{(1162.5)(-37.5)+(2375)(0)+(1187.5)(62.5)}{1162.5+2375+1187.5}\\ &= \frac{30625}{4725}\\ &= 6.481\,\text{mm}\end{aligned}$$

Comment

Step 11 of 25

Calculate the moment of inertia about y-axis

$$I_y = \frac{1}{12}b_1(h_1)^3 + A_1(\overline{z}+\overline{z}_1) + \frac{1}{12}h_2(b_2)^3 + A_2(\overline{z})^2 + \frac{1}{12}h_3(b_3)^3 + A_3(\overline{z}-\overline{z}_3)^2$$

Here, $\overline{z}$ is the location of centroid about z-axis, A_1 is the area of segment 1, A_2 is area of segment 2 and A_3 is area of segment 3.

Substitute $75\,\text{mm}$ for h_1, $15.5\,\text{mm}$ for b_1, $6.481\,\text{mm}$ for $\overline{z}$, $37.5\,\text{mm}$ for $\overline{z}_1$, $250\,\text{mm}$ for h_2, $9.5\,\text{mm}$ for b_2, $9.5\,\text{mm}$ for h_3, $125\,\text{mm}$ for b_3, $62.5\,\text{mm}$ for $\overline{z}_3$, $1162.5\,\text{mm}^2$ for A_1, $2375\,\text{mm}^2$ for A_2 and $1187.5\,\text{mm}^2$ for A_3.

$$\begin{aligned}I_y &= \begin{bmatrix}\left(\frac{1}{12}\right)(15.5)(75)^3+(1162.5)(6.481+37.5)^2+\frac{1}{12}(250)(9.5)^3+(2375)(6.481)^2\\ +\frac{1}{12}(9.5)(125)^3+(1187.5)(6.481-62.5)^2\end{bmatrix}\\ &= \begin{bmatrix}544921.875+2248554.465+1786.97917+99757.98237\\ +1146233.958+3725330.119\end{bmatrix}\\ I_y &= 8.18\times10^6\,\text{mm}^4\end{aligned}$$

Comment

Step 12 of 25

Calculate the moment of inertia about z axis

$$I_z = \frac{1}{12}h_1(b_1)^3 + A_1(\overline{y}+\overline{y}_1) + \frac{1}{12}b_2(h_2)^3 + A_2\left(\frac{h_2}{2}+\overline{y}\right)^2 + \frac{1}{12}b_3(h_3)^3 + A_3(\overline{y})^2$$

Here, $\overline{z}$ is the location of centroid about z-axis, A_1 is the area of segment 1, A_2 is area of segment 2 and A_3 is area of segment 3.

Substitute $75\,\text{mm}$ for h_1, $15.5\,\text{mm}$ for b_1, $-124.339\,\text{mm}$ for $\overline{y}$, $250\,\text{mm}$ for $\overline{y}$, $250\,\text{mm}$ for h_2, $9.5\,\text{mm}$ for b_2, $9.5\,\text{mm}$ for h_3, $125\,\text{mm}$ for b_3, $62.5\,\text{mm}$ for $\overline{z}_3$, $1162.5\,\text{mm}^2$ for A_1, $2375\,\text{mm}^2$ for A_2 and $1187.5\,\text{mm}^2$ for A_3.

$$\begin{aligned}I_z &= \begin{bmatrix}\left[\left(\frac{1}{12}\right)(75)(15.5)^3+(1162.5)(-124.339+250)^2\right]\\ +\left[\frac{1}{12}(9.5)(250)^3+(2375)\left(\frac{250}{2}+(-124.339)\right)^2\right]\\ +\left[\frac{1}{12}(125)(9.5)^3+(1187.5)(-124.339)^2\right]\end{bmatrix}\\ &= \begin{bmatrix}23274.22+18356673.55+12369791.67+1569.875\\ +8930.99+18358971.97\end{bmatrix}\\ &= 49.10\times10^6\,\text{mm}^4\end{aligned}$$

Comment

Step 13 of 25

Calculate the moment of inertia about yz-axis.

$$I_{yz} = A_1\left(\frac{\overline{y}_1}{2}\right)(-\overline{z}_1-\overline{z}) + A_2(-\overline{z})\left(\frac{h_2}{2}+\overline{y}\right) + A_3(-\overline{y})(\overline{z}_3-\overline{z})$$

Here, $\overline{y}$ is the location of centroid about y-axis, A_1 is the area of segment 1, A_2 is area of segment 2 and A_3 is area of segment 3.

Substitute $1162.5\,\text{mm}^2$ for A_1, $-37.5\,\text{mm}$ for $\overline{z}_1$, $2375\,\text{mm}^2$ for A_2, $0\,\text{mm}$ for $\overline{z}_2$, $1187.5\,\text{mm}^2$ for A_3 and $62.5\,\text{mm}$ for $\overline{z}_3$.

$$\begin{aligned}I_{yz} &= \begin{bmatrix}(1162.5)\left(-\frac{250}{2}\right)(-37.5-6.481)+(2375)(-6.481)\left(\frac{250}{2}-124.339\right)\\ +(1187.5)(-(-124.339))(62.5-6.481)\end{bmatrix}\\ &= 6390989.063+10174.36+8271348.9\\ &= 14.70\times10^6\,\text{mm}^4\end{aligned}$$

Comment

Step 14 of 25

Calculate the direction of principal axis.

$$\theta_p = \frac{1}{2}\tan^{-1}\left(\frac{-2I_{yz}}{I_y - I_z}\right)$$

Here, I_{yz} is moment of inertia about yz-axis, I_y is moment of inertia about y axis and I_z is moment of inertial about y axis.

Substitute $14.70\times10^6\,\text{mm}^4$ for I_{yz}, $49.10\times10^6\,\text{mm}^4$ for I_z and $8.18\times10^6\,\text{mm}^4$ for I_y.

$$\begin{aligned}\theta_p &= \frac{1}{2}\tan^{-1}\left(-\frac{(2)(14.70\times10^6)}{8.18\times10^6 - 49.10\times10^6}\right)\\ &= \frac{1}{2}\tan^{-1}(0.718475)\\ &= \frac{1}{2}(35.7)\\ &= 17.85^\circ\end{aligned}$$

Comment

Step 15 of 25

Calculate the principal of inertia about y axis.

$$I_{y'} = \left[\frac{I_y + I_z}{2} + \frac{I_y - I_z}{2}\cos 2\theta_p - I_{yz}\sin 2\theta_p\right]$$

Here, I_{yz} is moment of inertia about yz-axis, I_y is moment of inertia about y axis and I_z is moment of inertial about y axis.

Substitute $14.70\times10^6\,\text{mm}^4$ for I_{yz}, $49.10\times10^6\,\text{mm}^4$ for I_z and $8.18\times10^6\,\text{mm}^4$ for I_y, 17.85° for θ_p.

$$\begin{aligned}I_{y'} &= \left[\frac{8.18\times10^6+49.1\times10^6}{2} + \left\{\begin{matrix}\left(\frac{8.18\times10^6-49.1\times10^6}{2}\right)\times\cos(2\times17.85)\\ -14.7\times10^6\sin(2\times17.85)\end{matrix}\right\}\right]\\ &= (28.64-16.61523-8.578)\times10^6\\ &= 3.45\times10^6\,\text{mm}^4\end{aligned}$$

Comment

Step 16 of 25

Calculate the principal of inertia about z' axis.

$$I_{z'} = \left[\frac{I_z + I_y}{2} + \frac{I_z - I_y}{2}\cos 2\theta_p - I_{yz}\sin 2\theta_p\right]$$

Here, I_{yz} is moment of inertia about yz-axis, I_y is moment of inertia about y axis and I_z is moment of inertial about y axis.

Substitute $14.70\times10^6\,\text{mm}^4$ for I_{yz}, $49.10\times10^6\,\text{mm}^4$ for I_z and $8.18\times10^6\,\text{mm}^4$ for I_y, 17.85° for θ_p.

$$\begin{aligned}I_{z'} &= \left[\frac{8.18\times10^6+49.1\times10^6}{2} + \left\{\begin{matrix}\left(\frac{49.1\times10^6-8.18\times10^6}{2}\right)\times\cos(2\times17.85)\\ +14.7\times10^6\sin(2\times17.85)\end{matrix}\right\}\right]\\ &= (28.64+16.615+8.578)\times10^6\\ &= 53.85\times10^6\,\text{mm}^4\end{aligned}$$

Comments (1)

Step 17 of 25

Draw the schematic diagram of the section.

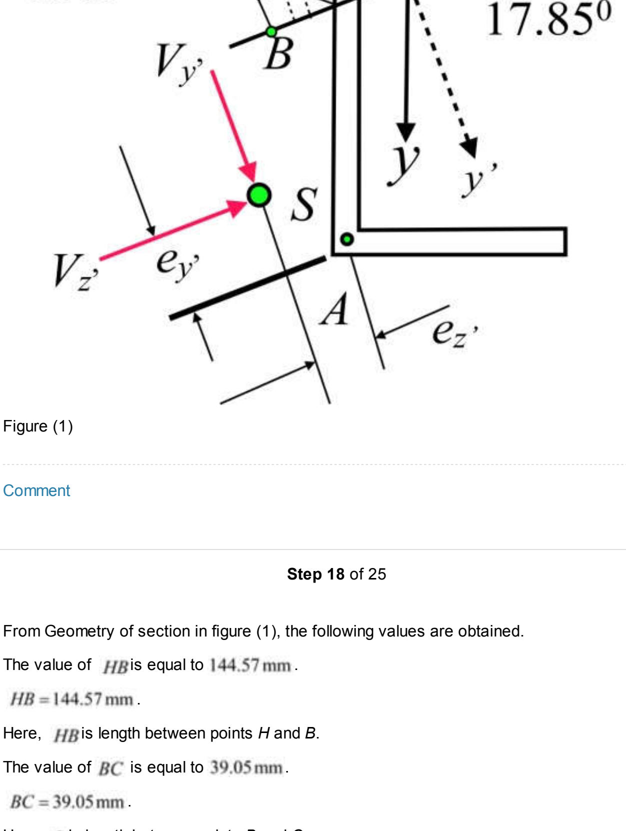


Figure (1)

Comment

Step 18 of 25

From Geometry of section in figure (1), the following values are obtained.

The value of HB is equal to $144.57\,\text{mm}$.

$HB = 144.57\,\text{mm}$.

Here, HB is length between points *H* and *B*.

The value of BC is equal to $39.05\,\text{mm}$.

$BC = 39.05\,\text{mm}$.

Here, B is length between points *B* and *C*.

Comment

Step 19 of 25

Write the relation for shear stress in xz direction.

$$T_{xz} = \frac{V_{y'}}{I_{z'}}\left[s(HB) - \frac{s}{2}\sin\theta_p\right]$$

Here, $V_{y'}$ is the shear in the y' plane, t is the thickness of segment 1, HB is length between *H* and *B* and *s* is the width of upper segment.

Substitute $53.85\times10^{-6}\,\text{m}^4$ for $I_{z'}$, $144.57\times10^{-3}\,\text{m}$ for HB and 17.85° for θ_p.

$$\begin{aligned}T_{xz} &= \frac{V_{y'}}{53.85\times10^{-6}}\left[s\left(144.5\times10^{-3} - \frac{s}{2}\sin(17.85^\circ)\right)\right]\\ &= \frac{V_{y'}}{53.85\times10^{-6}}\left[\left(0.1445s - \frac{s^2}{2}\sin 17.85^\circ\right)\right]\end{aligned}$$

Comment

Step 20 of 25

Due to the shear force $V_{y'}$:

Calculate the total shear force in the upper flange force F_1

$$F_1 = \int_0^s T_{xz}t\,ds$$

Here, T_{xz} is shear stress in xz-direction.

Substitute $\frac{V_{y'}}{53.85\times10^{-6}}\left(0.1445s - \frac{s^2}{2}\sin 17.85^\circ\right)$ for T_{xz}, $75\times10^{-3}\,\text{m}$ for s and $15.5\times10^{-3}\,\text{m}$ for t.

$$\begin{aligned}F_1 &= \int_0^{75\times10^{-3}\,\text{m}} \frac{V_{y'}(15.5\times10^{-3})}{53.85\times10^{-6}}\left[0.1445s - \frac{s^2}{2}\sin 17.85^\circ\right]ds\\ &= \frac{V_{y'}(0.0155)}{53.85\times10^{-6}}\left\{\left[0.1445\left(\frac{s^2}{2}\right)_0^{75\times10^{-3}}\right] - \left[\frac{1}{2}\left(\frac{s^3}{3}\right)\times\sin 17.85^\circ\right]_0^{75\times10^{-3}}\right\}\\ &= 287.8366V_{y'}\left[\frac{0.1445}{2}\times\left[(75\times10^{-3})^2 - 0^2\right]\right] - \left[\frac{1}{6}\times\sin 17.85^\circ\times\left[(75\times10^{-3})^3 - 0^3\right]\right]\end{aligned}$$

By solving preceding equation, the following relation is obtained.

Comment

Step 21 of 25

From the figure (1), write the relation shear force.

$$V_{y'}e_{z'} = (250\,\text{mm})(F_1)$$

Here, $e_{z'}$ is shear center S of the section z' plane and F_1 is total shear force in the upper flange.

Substitute $0.1108V_{y'}$ for F_1.

$$V_{y'}e_{z'} = (250\,\text{mm})(0.1108V_{y'})$$
$$e_{z'} = 27.7\,\text{mm}$$

Therefore, shear center S of the section about z' plane is $\boxed{27.7\,\text{mm}}$.

Comment

Step 22 of 25

Write the relation for shear stress in xz direction.

$$T_{xz} = \frac{V_{z'}}{I_{y'}}\left[s(BC) - \frac{s}{2}\cos\theta_p\right]$$

Here, $V_{z'}$ is the shear in the z' plane, t is the thickness of segment 1 and s is the width of upper segment.

Substitute $3.45\times10^{-6}\,\text{m}^4$ for $I_{y'}$, $39.05\times10^{-3}\,\text{m}$ for BC and $17.85°$ for θ_p.

$$\begin{aligned} T_{xz} &= \frac{V_{z'}}{3.45\times10^{-6}}\left[s\left(39.05\times10^{-3} - \frac{s}{2}\cos(17.85°)\right)\right] \\ &= \frac{V_{z'}}{3.45\times10^{-6}}\left[\left(0.03905s - \frac{s^2}{2}\cos17.85°\right)\right] \end{aligned}$$

Comment

Step 23 of 25

Due to the shear force $V_{z'}$:

Calculate the total shear force in the upper flange force F_1

$$F_1 = \int_0^s T_{xz}t\,ds$$

Here, T_{xz} is shear stress in xz-direction.

Substitute $\frac{V_{z'}}{3.45\times10^{-6}}\left[\left(0.03905s - \frac{s^2}{2}\cos17.85°\right)\right]$ for T_{xz}, $75\times10^{-3}\,\text{m}$ for s and $15.5\times10^{-3}\,\text{m}$ for t.

$$\begin{aligned} F_1 &= \int_0^{75\times10^{-3}\,\text{m}} \frac{V_{z'}(15.5\times10^{-3})}{3.45\times10^{-6}}\left[\left[\left(0.03905s - \frac{s^2}{2}\cos17.85°\right)\right]\right]ds \\ &= \frac{V_{z'}(0.0155)}{3.45\times10^{-6}}\left\{\left[0.03905\left(\frac{s^2}{2}\right)_0^{75\times10^{-3}}\right] - \frac{1}{2}\left(\frac{s^3}{3}\right)\times s\cos17.85°\Big]_0^{75\times10^{-3}}\right\} \\ &= 4492.75V_{z'}\left[\frac{0.03905}{2}\times\left[(75\times10^{-3})^2 - 0^2\right]\right] - \left[\frac{1}{6}\times\cos17.85°\times\left[(75\times10^{-3})^3 - 0^3\right]\right] \end{aligned}$$

Comment

Step 24 of 25

By solving preceding equation, the following relation is obtained.

$$\begin{aligned} F_1 &= 4492.75V_{z'}\times\left[1.09828\times10^{-4} - 6.693\times10^{-5}\right] \\ &= 0.1928V_{z'} \end{aligned}$$

Comment

Step 25 of 25

From the figure (1), write the relation shear force.

$$V_{z'}e_{y'} = (250\,\text{mm})(F_1)$$

Here, $e_{y'}$ is shear center S of the section y' plane and F_1 is total shear force in the upper flange.

Substitute $0.1928V_{z'}$ for F_1.

$$V_{y'}e_{z'} = (250\,\text{mm})(0.1928V_{y'})$$
$$e_{z'} = 48.2\,\text{mm}$$

Therefore, shear center S of the section about y' plane is $\boxed{48.2\,\text{mm}}$.

Comment